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Procedure for Constructing the Ship Motion Model and Synthesizing Model-Based Control Algorithms

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Abstract: The paper proposes a technique for constructing the simulation model of a surface ship motion based on the specified composition and parameters of actuators. As an example, constructing the simulation model of a car and train ferry is considered. Based on the model, control algorithms have been developed for stabilizing the longitudinal component of the ship linear velocity and heading stabilization.

Keywords: dynamic model, maneuvering performance, disturbances, control inputs.

1. INTRODUCTION

Simulation models of vehicles are traditionally applied to develop the control laws in different motion modes. They are also important in developing the simulators employed to test the interaction between the navigation and control systems and to train the navigators. Clearly, the simulation model used both for the control law synthesis and training should be adequate to the ship real motion. The ship motion model is selected based on the tasks performed by the ship, and, therefore, on the motion control and navigation data processing tasks. As an adequacy criterion of the simulation model, it is rational to consider the agreement between the generated navigation and dynamic parameters and the ship real motion with account for the wind, waves, and current. Then the initial data for the motion model are the braking and acceleration abilities, and maneuvering (circulation) performance obtained during the ship sea trials and recorded in the maneuvering booklet (MB) according to the International Maritime Organization (IMO) resolutions [1–3], and the requirements of the Russian Maritime Register of Shipping [4].

The problem is mostly solved by presenting the ship dynamics as simplified models corresponding to the experimental data and approximately describing the ship motion in the horizontal plane [5–7]. In [8–12] the simplified models are constructed using special heuristic formulas for the ship characteristic parameters. This approach, however, fails to provide a comprehensive idea of the ship's behavior in different motion modes such as mooring and dynamic positioning mode, and to verify the control algorithms.

The paper proposes a procedure for constructing the five-degrees-of-freedom model of a surface ship and presents the model development principles. The model is used to simulate the navigation parameters from the inertial navigation system (INS), log, GNSS receiver, which determines the model order [13–18]. The ship is disturbed by the action of wind, waves, and current.

As an example, we consider constructing the motion model of a CNF19M car and train ferry (Fig. 1).

Fig. 1. Appearance of the CNF19M ferry.

Its main parameters are [19]:

- length – 199.9 m;
- width – 27.4 m;
- displacement – 21647 tons;
- draught – 6 m;
- operation speed – 16.5 kt.

The paper contains four main sections. The first section presents the initial data for modeling – configuration of the propulsion/steering unit and the contents of the MB. The second section describes the ship motion model. The model is a system of five nonlinear differential equations in the body frame and five kinematic relationships determining the ship attitude in the body frame and the ship path in the geographical frame. The third section details the proposed procedure for determining the model unknown coefficients. Finally, in the fourth section the obtained ship motion model is used to generate the control law stabilizing the longitudinal component of the ship velocity and heading, and modeling results under disturbances are presented.

2. INITIAL DATA FOR MODEL CONSTRUCTION

To construct the simulation model, the propulsion/steering unit was presented with the following actuators:

- two main engines with oppositely rotating propellers and controllable pitch propellers (CPP), i.e., the speed is controlled by rotating the propeller blades;
- two bow steering propellers (SP) and one stern SP generating the thrust force by changing the propeller spinning rate;
- steering device (SD) with two rudders behind each propeller.

The motion model parameters are determined based on the main reference data from the MB containing:

- stopping and acceleration performance;
- typical circulations of the real ship under different rudder angles;
- performance of the SP.

The disturbances (wind, waves, and current) are generated using the models given in [13, 20].

3. DESCRIPTION OF MOTION MODEL

We introduce the following reference frames:

- body (ship-fixed) frame *Oxyz* with the origin *O* at the center of gravity, axis *Ox* is directed to the bow, axis *Oy* is directed to the starboard, and axis *Oz* points upwards from the deck plane;
- geographical frame *ONE* (axis *ON* points to the North, axis *OE* points to the East), the origin is taken to be some initial point, for example, of the loxodromic part of the trajectory.

The Euler's angles describing *Oxyz* attitude relative to *ONE* are denoted as follows: ψ – yaw, γ – roll, ϑ – pitch. Then, $\psi = K - K^{set}$, where K is the ship heading, K^{set} is the relative bearing of the loxodromic part. The roll to starboard, pitch to the stern, and yaw to starboard are positive.

Traditional assumptions from the theory of ship oscillations [13–16] are used in the modeling:

- the ship is symmetrical relative to the centerline plane both for the hull and mass distribution, and the main inertia axes coincide with the coordinate axes;
- the pressure at each point of the submerged surface of the ship body is the same as at the relevant wave point;
- hydrodynamic forces acting on the ship can be grouped into potential and viscous;
- incident waves have a small slope angle.

Considering the ship motion in the geographical frame we take that the ship center of mass moves nearly horizontally, i.e., vertical coordinate and velocity are assumed zero.

Then the motion model in the body frame is a system of five nonlinear differential equations describing the ship dynamics [13–16]:

$$
\begin{gathered}
(m+\lambda_{11})\dot{V}_x = (m+\lambda_{22})V_y\omega_z - R_x + T_x + f_1\left(V^2 + L_r^2\omega_z^2\right)\delta^2 + F_{ax}, \\
(m+\lambda_{22})\dot{V}_y = a_{11}VV_y + a_{12}V\omega_z + n_{11}V_y\left|V_y\right| + n_{12}\left|V_y\omega_z\right| + f_2V^2\delta + T_{\text{SP}y} + F_{ay} + F_{wy}, \\
(J_{xx}+\lambda_{44})\dot{\omega}_x = \mu_4\omega_x - mgh_0\mathrm{J} + M_{\text{SP}x} + M_{ax} + M_{wx}, \\
(J_{yy}+\lambda_{55})\dot{\omega}_y = \mu_5\omega_y - mgH_0\psi + M_{wy}, \\
(J_{zz}+\lambda_{66})\dot{\omega}_z = a_{21}VV_y + a_{22}V\omega_z + f_6V^2\delta + M_{\text{SP}z} + M_{az} + M_{wz},
\end{gathered} \tag{1}
$$

where V_x, V_y are the longitudinal and lateral components of the ship velocity; V is the modulus of the velocity vector; ω_x, ω_y, ω_z are the projections of the ship angular velocity vector on the body frame; m is the ship mass; L_r is the distance between the center of gravity and the ship rudder; g is the gravity acceleration; h_0 is the transverse metacentric height; H_0 is the longitudinal metacentric height; J_{xx}, J_{yy}, J_{zz} are the ship moments of inertia relative to the body frame; λ_{ii} are the added masses and moments of inertia relative to the principal axes; T_x is the propeller thrust force; R_x is the ship body drag force; δ is the rudder angle; F_{ax}, F_{ay}, M_{ax}, M_{az} are the projections of aerodynamic forces and moments on the relevant axes; F_{wy}, M_{wx}, M_{wy}, M_{wz} are the projections of forces and moments due to waves; T_{SPy}, M_{SPx}, M_{SPz} are the projections of forces and moments due to SP; f_i, a_{ij}, n_{ij} are the sought hydrodynamic coefficients; μ_i are the sought parameters of roll and pitch decay.

The rudder angles are considered to be the same, and the effect of deflecting two rudders is simulated with one force and moment.

We also introduce five kinematic relationships determining the ship attitude in the body frame and its trajectory in the geographical frame:

$$R_\varphi \dot{\varphi} = V_x \cos K - V_y \sin K + V_c \cos K_c,$$

$$R_w \dot{\lambda} \cos\varphi = V_x \sin K + V_y \cos K + V_c \sin K_c,$$

$$\dot{\gamma} = \omega_x,$$

$$\dot{\vartheta} = \omega_y,$$

$$\dot{\psi} = \omega_z,$$

where φ, λ are the geographical coordinates (latitude and longitude); R_φ, R_w are the current curvature radii of the meridian and prime vertical; V_c, K_c are the speed and direction of current.

4. DETERMINING THE MODEL PARAMETERS

The model is constructed in several steps. Seven main steps can be distinguished. Their sequence corresponds to the ship development steps: at first performance data are specified, then the ship controllability is tested, and finally, its seagoing ability under wind, waves, and current is analyzed.

At the first step we consider the analytical dependency of propeller thrust force on the propeller pitch ratio P/D, longitudinal component of speed through the water V_x corresponding to this ratio, CPP spin rate n и diameter D [13]:

$$T_x = K_{T1}P/D + K_{T2}V_x(1 - w),$$

$$K_{T1} = K_1\rho_w|n^3|D^4,\ K_{T2} = -K_2\rho_w n^2 D^3,$$

where $P/D = \pi\,\mathrm{tg}\,\alpha$, α is the CPP blade angle (blade angles of port and starboard CPP are taken to be the same); w is the wake factor; ρ_w is the water density; n is the propeller spin rate; D is the propeller diameter. Two unknown coefficients of the CPP thrust force K_1, K_2 are determined using the available force T_x [21] under the fixed values of the given parameters. The coefficients are computed using the least squares method.

At the second step, the initial (corresponding to zero drift angle) drag factor C_{x0} is computed based on the analytical dependency of the ship body drag force R_x on the speed through the water, wetted area surface Ω_m, drift angle β (throughout its range) [13], and available R_x under the fixed values of parameters [21]. The dependency is given by [13]

$$R_x = C_x\,\rho_w V_x^2/2\cdot\Omega_m,$$

where

$$C_x = 0.075\sin((\pi - \arcsin(C_{x0}/0.075)) \times \times (1 - 0.4775|\beta|)).$$

The value of C_{x0} is also computed using the least squares method. It can be updated based on acceleration and stopping performance obtained during acceleration and deceleration and recorded in the MB.

The model data are compared with the real ferry acceleration and stopping modes in Fig. 2. The ranges, duration, initial and final speeds for each mode are taken from the ship maneuvering performance table.

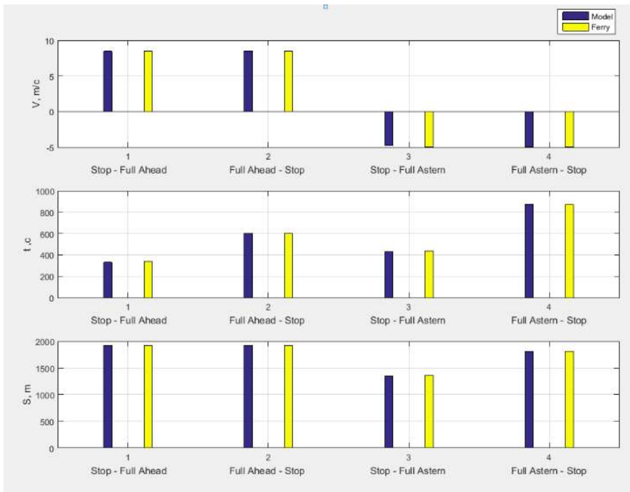

Fig. 2. Ferry acceleration and stopping performance.

At the third step, lateral motion equations are considered (the second and fifth equations in (1)). The coefficients C_i^j under the hydrodynamic forces and moments included in the lateral motion equations [13–17] are computed based on the known design characteristics (length, width, draught, displacement volume, etc.) using empirical formulas [17], nomographs [13], or analogs.

The sought parameters are six hydrodynamic coefficients:

$$\begin{aligned}
a_{11} &= -C_y^{\beta}\rho_w A_S/2,\\
a_{12} &= C_y^{\omega}\rho_w A_S/2 - m - \lambda_{11},\\
a_{21} &= -C_m^{\beta}\rho_w L A_S/2 - \lambda_{11} - \lambda_{22},\\
a_{22} &= C_m^{\omega}\rho_w L A_S/2,\\
n_{11} &= -C_y^{\beta\beta}\rho_w A_S/2,\\
n_{12} &= -C_y^{\beta\omega}\rho_w L A_S/2,
\end{aligned} \tag{2}$$

where A_S is the midship area; L is the ship length.

At this step the coefficients f_i are also determined, which account for the effect of rudder angles on the circulation parameters. The coefficients should be selected based on maximum agreement between the model trajectories and MB data (Fig. 3). It is recommended to use the technique proposed in [22]. The coefficients determined according to (2) are updated using the similar technique.

At the fourth step decay coefficients of roll and pitch components μ_I are determined. Dimensionless decay coefficients are determined as provided by expected initial roll and pitch responses. Further they may be recalculated into the sought coefficients μ_I with account for the motion natural frequencies determined by the parameters from the third and fourth equations in (1).

At the fifth step, wave disturbances are considered. The unknown parameters here are the coefficients included in the formulas for the projections of wave forces and moments:

$$\begin{aligned}
F_{wy} &= -f_{wy} g \alpha_w,\\
M_{wx} &= f_{wm} m g h_0 \alpha_w,\\
M_{wy} &= f_{wm} m g H_0 \alpha_w,\\
M_{wz} &= -f_{wmz}\frac{\omega_w^2}{\omega_{im}}\dot{\alpha}_w,
\end{aligned}$$

where α_w is the wave slope angle described by the second-order Markov process; ω_w is the dominant wave frequency depending on the sea state, ω_{im} is the apparent wave frequency; h_0 is the transverse metacentric height; H_0 is the longitudinal metacentric height; f_{wy}, f_{wm}, f_{wmz} are the sought reduction coefficients. The unknown parameters are estimated so that to obtain the roll and pitch amplitudes corresponding to the specified sea state.

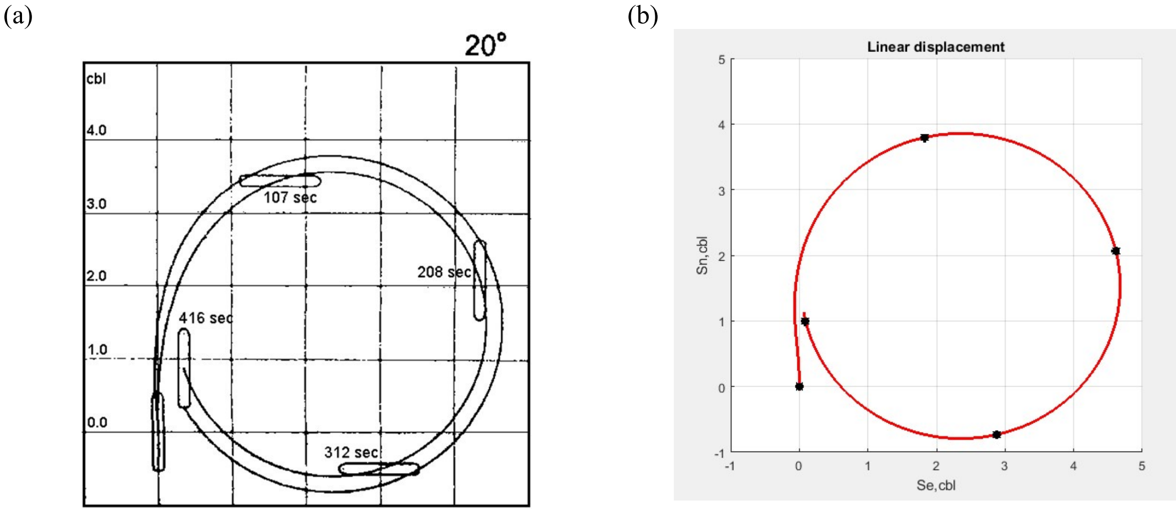

Fig. 3. Circulation with rudder angle δ=20°: (*a*) fragment of maneuvering performance table; (*b*) modeling results (coordinates at the time of rotation by 90°, 180°, 270°, 360° are shown with black dots).

The wave slope angle can be determined using the stochastic model of wave disturbance given in [23]. In the simplest case, a model of regular waves in the form of sinusoidal process can be accepted.

At the sixth step, effect of wind disturbances is considered. The formulas for wind-driven aerodynamic forces and moments are given by [13]:

$$\begin{aligned}
F_{ax} &= -f_{ax}\rho_a A_{vp} V_k\left(-V_x\cos\beta + V_w\cos(K_w - K)\right),\\
F_{ay} &= -f_{ay}\rho_a A_{vl} V_k\left(V_x\sin\beta + V_w\sin(K_w - K)\right),\\
M_{ax} &= f_{am}\rho_a A_{vl} V_k\left(V_x\sin\beta + V_w\sin(K_w - K)\right)Z_p,\\
M_{az} &= f_{am}\rho_a A_{vl} V_k\left(V_x\sin\beta + V_w\sin(K_w - K)\right)\times\\
&\times\left(0.075L + b_m - |\gamma_1| L/2\pi\right),
\end{aligned}$$

where V_w, K_w is the wind speed and direction; ρ_a is the air density; A_{vp}, A_{vl} are the front and side wind areas; Z_p is the vertical distance between the center of wind area and the center of gravity; b_m is the mean distance between the center of gravity and the center of wind area along the longitudinal axis; V_k, γ_1 is the speed and direction of the apparent wind, f_{ax}, f_{ay}, f_{am} are the sought coefficients. Unknown parameters are estimated so that the trajectory during circulation corresponded to the wind drift under the specified sea state.

At the seventh step, the forces and moments induced by the SP are considered. The resulting force T_{SP} is formed by summing the forces T_{SPi}, generated by each SP according to the relationship

$$T_{SPi} = k_{SPi}\, n_{SPi}\, |n_{SPi}|,$$

where n_{SPi} is spin rate of the propeller of the relevant SP, k_{SPi} is the proportionality factor. SP induced moments are generated with account for the arms of the relevant forces equal to the distances between the gravity center and the SP.

The coefficients k_{SPi} are updated based on the discrepancy between MB reference data and the modeling data obtained with the same control signals n_{SPi}. We can also consider the factor dependency on the current longitudinal and transverse components of the linear velocity and heading variation rate.

The proposed procedure applied to construct the mathematical motion model for a car and train ferry provided good agreement between the motion parameters and MB parameters. Discrepancies between the model and MB data were found in:

- ship coordinates during the circulations at the specified times with different rudder angles (relative to the circulation diameter);
- traveled distance during acceleration and stopping.

The discrepancy did not exceed 5–8%, which is acceptable.

5. USING THE MOTION MODEL TO GENERATE CONTROL INPUTS

We formulate the ship motion control laws relating to generation of the required thrust force and SD force based on the constructed motion model. Consider two problems: stabilizing the velocity longitudinal component and heading stabilization.

The ship motion control system is given in detail in Fig. 4.

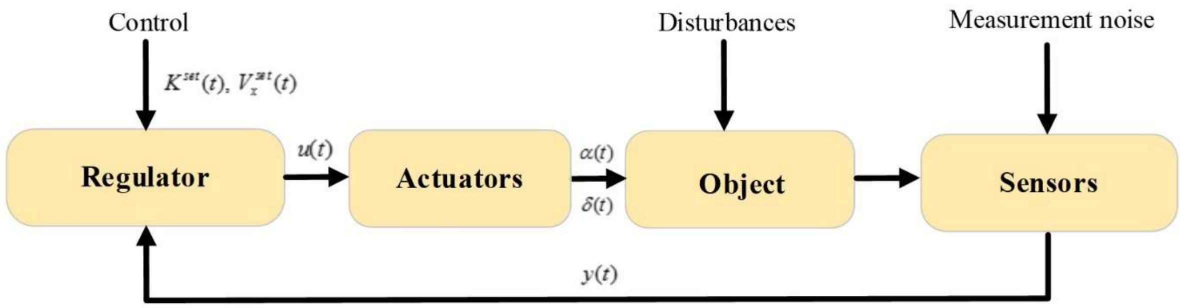

Fig. 4. Ship heading and speed stabilization system.

The specified speed V_x^{set} is stabilized by rotating the CPP blades by angle $\alpha(t)$ using the PI regulator. The specified heading K^{set} is stabilized by deflecting the rudder by angle $\delta(t)$ using the PID regulator. The regulators' coefficients have been determined using the procedure given in [24].

The actuators to which the control inputs from the regulators are fed are aperiodic links with the known time constants.

The following constraints on the maximum blade deflection angle and maximum angle rate are imposed when generating the output signals:

$$|\alpha(t)| \le \alpha_m,\ |\dot{\alpha}(t)| \le \dot{\alpha}_m,$$
$$|\delta(t)| \le \delta_m,\ |\dot{\delta}(t)| \le \dot{\delta}_m;$$

In the simulation, $\alpha_m = 35°$, $\dot{\alpha}_m = 1.5°/s$, $\delta_m = 25°$, $\dot{\delta}_m = 2.5°/s$.

The measured navigation parameters fed to the regulators are modeled by considering the additive noise of the sensors (log and inertial navigation system with navigation grade sensors). The log error is modeled as an exponentially correlated random process with the specified RMS error and correlation interval. The heading error ΔK is described with the following approximate model:

$$\Delta\dot{\omega}_z = -\alpha_\omega \Delta\omega_z + w_1,$$

$$\Delta\dot{K} = \Delta\omega_K,$$

$$\Delta\dot{\omega}_K = -\beta_K^2 \Delta K - 2\alpha_K \Delta\omega_K + k\Delta\omega_z + w_2,$$

where $\Delta\omega_z$ is the angular rate error described with an exponentially correlated process with the correlation interval $1/\alpha_\omega$; $\Delta\omega_K$ is the auxiliary parameter describing the Schuler oscillations with circular frequency β_K in the heading error; α_K is the magnitude inversely proportional to the correlation interval of the heading error; k is the factor describing the cross-correlation between angular rate and heading errors; w_i are the generating white noises noncorrelated with each other.

At the first synthesis step, the control is generated for a motion model free of external disturbances. The regulator coefficients are selected so that to minimize the transient time with no overshoots. At the second step, external disturbances were considered, and the control law factors were empirically corrected to improve the heading stability during motion at a specified speed.

The plots of the CPP blade angle change, blade spin rate, and longitudinal and transverse components of linear velocity at the ship acceleration from $V_x = 5$ m/s to 10 m/s and further stabilization at the specified speed are presented in Fig. 5. These results have been obtained for Beaufort number 5; $K_w = 180°$, specified heading is 10°. Simulation was performed in MATLAB 2018b.

Figure 6 shows the plots of the rudder angle, SD deflection rate, heading and measured heading rate during the 90° turn and further heading stabilization.

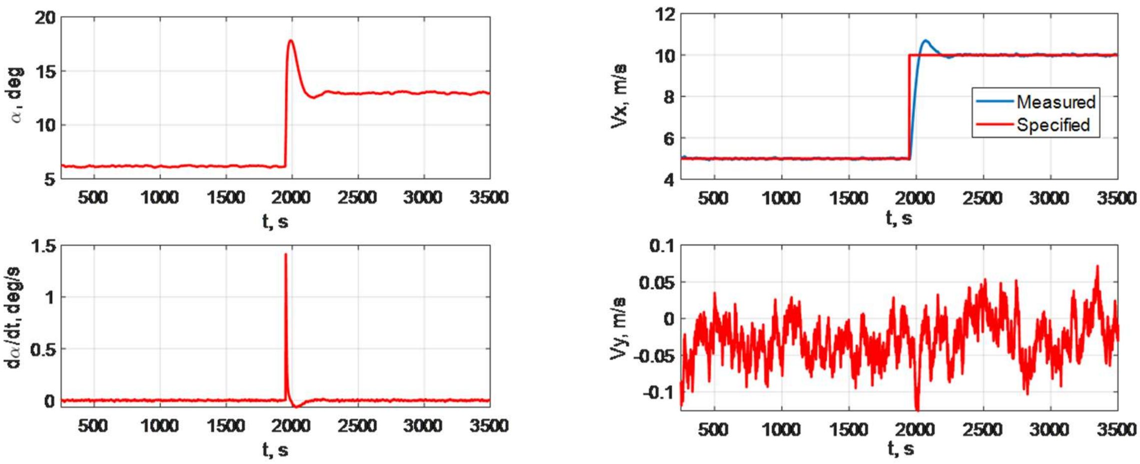

Fig. 5. Simulation of the motion control system when achieving the specified speed and speed stabilization

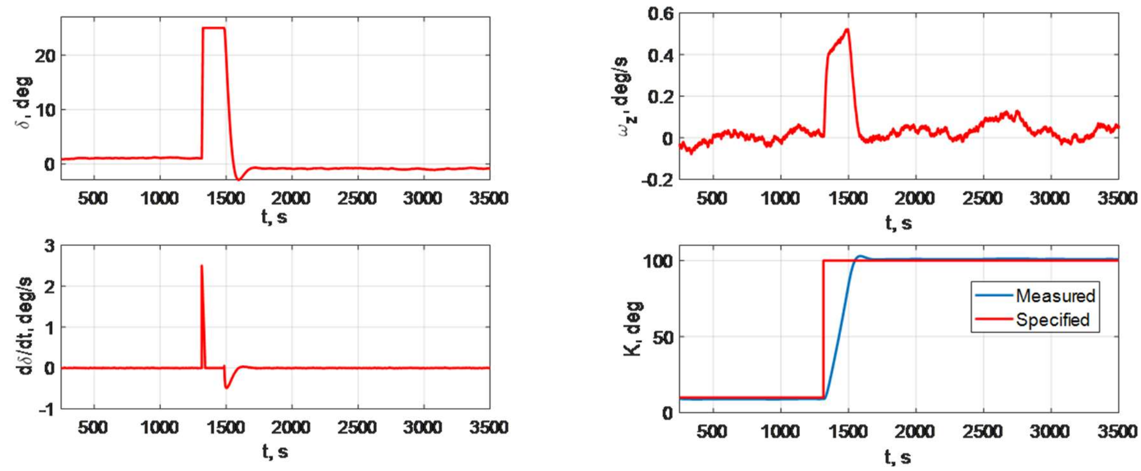

Fig. 6. Simulation of the motion control system when achieving the specified heading and heading stabilization

The following performance characteristics of the control system under wind and waves, which correspond to Beaufort number 5 or less, have been obtained:

- maneuvering time to change the heading by 90° – 4 min;
- transient time when changing the speed from 5 to 10 m/s under the fixed spin rate of main engine shaft – 3 min.

6. CONCLUSIONS

The proposed procedure for constructing the simulation model of a surface ship motion uses the data from the maneuvering booklet. It ensures the construction of a complete dynamic model sufficiently adequate to the ship real motion and generating a full set of navigation and dynamic parameters needed, in particular, to simulate the operation of navigation systems. The model has been employed to generate the control law providing the stabilization of the speed longitudinal component and heading. The synthesized control laws have been shown to be efficient under external disturbances.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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